Fast Eddy Current Simulation in Thick Split Cylinders of Finite Length Induced by Coils of Arbitrary Geometry

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Abstract — Eddy currents are inevitably induced when time-varying magnetic field gradients interact with the metallic structures of a magnetic resonance imaging (MRI) scanner. The secondary magnetic field produced by this induced current degrades the spatial and temporal performance of the primary field generated by the gradient coils. Although these undesired effects can be minimized a residual eddy current still remains as an unsolved problem in MRI. Accurate simulation of these eddy currents is important in the design of gradient coils and cryostat vessels. The approach presented in this paper divides thick conducting cylinders into thin layers (thinner than the skin depth) and expresses the current density on each as a Fourier series. The coupling between each mode of the Fourier series with every other is modeled with an inductive network method. The currents induced by a split actively shielded x-gradient coil were simulated assuming a finite length split cylindrical cryostat. The new method is a valuable tool to understand the mechanism of interaction between hybrid imaging modalities such as positron emission tomography (PET) and MRI.

I. INTRODUCTION

In Magnetic Resonance Imaging (MRI) the signal encoding is performed by a rapidly changing magnetic field produced by a set of three gradient coils conveniently arranged to generate a highly linear magnetic field along the *x*, *y* and *z*-coordinates. When a gradient coil is switched on or off, eddy currents are induced in the highly conducting shields and dewar walls that contain and support the superconducting magnet. In turn, a secondary magnetic field is created that opposes and interferes with the desired gradient field (the primary field). This effect results in a miss-location of the signal and consequential image distortion [1]. Although, these inconvenient eddy current effects have been greatly reduced by the application and combination of active/passive shielding and current pre-emphasis [1], residual but significant eddy currents still remain as an unsolved problem in MRI. A variety of integral and differential methods have been presented to evaluate the eddy currents induced by gradient coils of planar and cylindrical geometries. FEM codes permit the simulation of a realistic cryostat but with a costly computational effort [2].

In this paper we have extended the network method [3] to a more general formulation which includes axially-split thick cylinders of finite length. We applied the infinitesimal thin shell approach where a surface current density, expressed as a finite Fourier series, flows in each cylindrical surface of radius ρ, conductivity σh and axial length L (h is the shell thickness). Boundaries conditions at edges L and L_1 are specified in such a manner that no currents flows along the axial direction out of the conducting domain. The method is validated and details of the current induced by *x*-gradient coil are presented and discussed. The new technique can be apply in the analysis of the interaction mechanism between hybrid modalities such as PET-MRI or guide therapy-MRI.

II. MATERIAL AND METHODS

Figure 1, shows a non-magnetic split cylinder made of a linear isotropic conducting media of conductivity σ (region V_i) and immerse in an external magnetic field created by the known source $\mathbf{J}_s(\mathbf{r},t)$ excited at t=0. $\mathbf{J}_s(\mathbf{r},t)$ is immersed in the medium of σ=0 and $\mu_0=4\pi\times10^{-7}$ H/m.

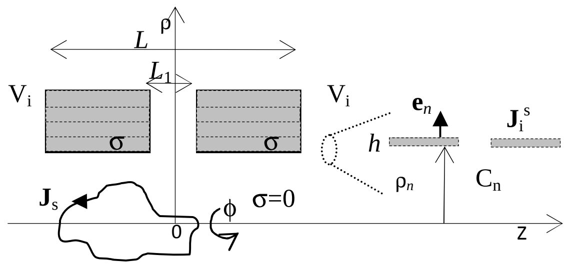

Fig. 1. Half cross-section of the split cylinder. No current flows in the axial gap of length L_1.

It is assumed that no displacement currents exist. The conducting cylinder V_i is divided in N layers of thickness h, where h is much smaller than the skin depth δ. The axial length of the shell L and radius ρ_n are much larger than the thickness h. The axial gap L_1 is smaller than the shell total length L. The conducting layers, C_n, are treated as thin shells of surface conductivity σh. We assume that the current flowing in the radial direction is nearly zero ($J_\rho\approx0$), hence no resistive coupling exists between shells, but they are inductively coupled. The induced current density has the form $\mathbf{J}_i^s(z, \phi, \rho, t)=[J_\phi(z, \phi, t)\mathbf{e}_\phi+J_z(z, \phi, t)\mathbf{e}_z]\delta(\rho-\rho_n)$, wehere δ is the delta function. In order to consider all possible spatial variations of the current density $\mathbf{J}_s(\mathbf{r},t)$ representing the exciting coil, $\mathbf{J}_i^s(\mathbf{r},t)$ is expressed as a finite Fourier series of orthogonal functions [3]. In this work we enforce the condition $J_z(z=\pm L_1/2, \phi, t)=0$ to guarantee that no current flows out of the conducting domain, hence $\nabla\cdot\mathbf{J}_i^s(\mathbf{r},t)=0$ holds. Using a proper Green's function expansion in cylindrical coordinates the Fourier transform of $J_\phi(\mathbf{r}, t)$ can be obtained. Power dissipation and magnetic energy are expressed in Fourier space using modified Bessel functions [4]. In this manner we avoid the costly 3D full evaluation of theses parameters [3]. The time-harmonic solution of the diffusion equation can be written as

$$\begin{pmatrix} \mathbf{Z}_{ii} \\ \mathbf{A} \end{pmatrix} \mathbf{s}_i(\omega) = \begin{pmatrix} -i\omega \mathbf{M}_{is} \\ \mathbf{B} \end{pmatrix} \qquad (1)$$

where $\mathbf{Z}_{ii}=(i\omega\mathbf{M}_{ii}+\mathbf{R}_{ii})$, $\mathbf{M}_{is}$ is the mutual inductive coupling between the source and the conducting shells and $\mathbf{M}_{ii}$ and $\mathbf{R}_{ii}$ are the self inductive and resistive coupling of the conducting shells, respectively [3]. The matrix **A** contains the contribution of all the modes included in $J_z(z=\pm L_1/2, \phi)$ at the edge $z=\pm L_1/2$ evaluated along the ϕ-direction. The vector **B** is populated with zeros. Details of the transient solution can be found in [5].

III. Results and Discussions

The method was validated against commercial software FEMLAB using a canonical problem. Figure 2 demonstrates the accuracy of the new method when predicting the skin depth of the currents induced by a double-circular loop driven with a time-harmonic current variation.

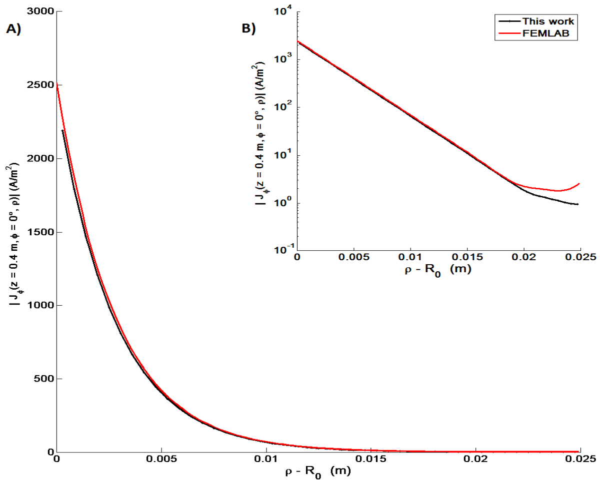

Fig. 2. Eddy current penetration into a split, thick, conducting cylinder versus radial distance at z=40 cm. The thickness was set to 25 mm with 20 cm of gap. The axial length was set to 1.4 m and the radius to 50 cm. The coil radius was 40 cm. B). Logarithmic values of the current density.

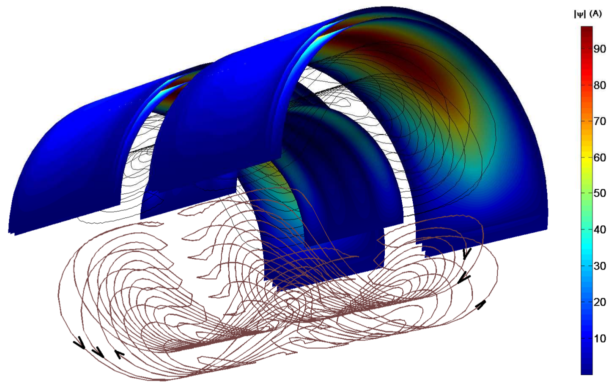

Fig. 3. Split gradient coil and a representation of the absolute value of the current induced in each cylinder. The arrow shows the current direction.

The new method predicted a skin depth of 2.785 mm, which is close to δ=2.8 mm at 1 kHz and $\sigma=32.26\cdot10^6$ S/m. predicted by FEMLAB Figure 3, shows a representation per cylinder of the stream function of the current density induced by a split *x*-gradient coil in a split cryostat vessel and RF copper shield.

The cryostat dimensions and conductivity of each cylinder can be found in [6], but we have included an axial gap of 20 cm. The *x*-gradient coil produces a gradient field 10 mT/m and has an axial gap of 20 cm. The simulation was completed in 25 min. A similar model will require a huge computational burden and computing time if performed using a FEM code. The RF shield and the warm bore cylinder produced the largest power. We found that mostly all the zonal harmonics that might shift and destroy the B_0 spatial profile are originated in the RF-shield.

IV. Conclusions

The new method is able to accurately predict the skin depth in split cylinders excited by coils of arbitrary geometry. The inclusion of the boundary conditions in the solution of the diffusion equation fulfills the continuity equation of the current density. The new method is a valuable tool to understand eddy current interaction mechanism in hybrid MRI systems and thus minimize the undesired interactions between the two multimodal imaging techniques.

V. References

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